A Rigorous and Efficient Full-Wave Analysis of Uniform Bends in Rectangular Waveguide Under Arbitrary Incidence

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***Abstract*—In this paper, a rigorous full-wave analysis of uniform bends in rectangular waveguide is performed. An accurate and efficient method-of-moments solution combined with the generalized-admittance-matrix (GAM) formulation is proposed in order to achieve a full-wave characterization of the analyzed structures. This full-wave modal solution turns out to be necessary for modeling complex microwave devices involving an arbitrary number of discontinuities between curved and straight waveguides, where all the modes of the involved guides are excited. The key feature of the presented method lies in the GAM representation of single and cascaded curved E- and H-plane uniform bends, which allows to construct accurate models of the investigated discontinuities. To validate the theory, the convergence of the method is discussed and comparisons between our simulations and theoretical and experimental data are presented. The excellent behavior of our results, together with the computational efficiency of the proposed method, proves that the developed computer-aided-design tool can be successfully used in the design of complex microwave subsystems involving curved waveguides.**

***Index Terms*—Generalized-admittance-matrix (GAM) representation, method of moments, rectangular waveguides, uniform bends.**

I. Introduction

THE modal analysis of uniform bends in rectangular waveguide has been subject to investigation from many researchers since these components are essential and very frequently used in sophisticated microwave devices for both space and ground applications (e.g., diplexers, multiplexers, radar seekers, beam-forming networks, satellite communication systems, etc.). In order to overcome the mechanical constraints of such systems demanded by the industry requirements, it is often necessary to resort to compact bends. A full-wave characterization of these structures is therefore crucial to develop efficient computer-aided design (CAD) tools for the analysis and design of arbitrary microwave devices involving circular bends.

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In this way, the development of accurate and efficient methods for the analysis of the above-mentioned uniform bends has received considerable attention in the technical literature. In 1948, Rice [1] employed matrix theoretical techniques to obtain approximate formulas for the reflection coefficient of E- and H-plane circular bends and, in 1966, Cochran and Pecina [2] expanded the radial component of the electric and magnetic fields in terms of Bessel functions in order to obtain the angular propagation constants of curved regions. By means of a method based on an integral-equation formulation, Bates [3] and Mittra [4] analyzed the junction between straight and curved waveguides, and Lewin *et al.* [5] investigated E- and H-plane bends with a method based on a perturbational analysis. More recently, a mode-matching technique for modeling discontinuities involving uniform bends was proposed by Accatino and Bertin [6] and Weisshaar *et al.* [7], and a multimodal method for analyzing full-band matched waveguide bends was outlined by Mongiardo *et al.* [8]. Other different techniques have been also investigated: equivalent circuits based on lumped elements were proposed by Carle [9] and Marcuvitz [10], Gimeno and Guglielmi [11] have efficiently used a multimode equivalent-circuit representation for the analysis of bends, Pregla [12] has presented a procedure for the analysis of concatenations of straight and curved waveguides based on the method of lines, and Cornet *et al.* [13] have obtained the scattering matrix of uniform bends with a technique based on differential geometry. However, the aforementioned works only consider the excitation due to the fundamental mode of the rectangular input port, thus dividing the analysis into H- and E-plane bends, both represented in Fig. 1, where a and b are the dimensions of the rectangular guide ($a > b$). Therefore, if the full-wave spectrum was excited into the rectangular input port, all these previous analysis techniques could not be used in order to analyze the considered device.

Thus, the main objective of this paper is to introduce a full-wave analysis procedure based on the generalized admittance matrix (GAM) representation [14] of H- and E-plane bends, which allows to consider the possible incidence of any arbitrary mode of the rectangular input port. The GAM formulation allows to model the junction between straight and curved waveguides leading to simple analytical expressions. The proposed method, combined with an efficient inversion technique for solving banded linear equation systems [15], achieves convergent and accurate results and shows excellent agreement with experimental data. Moreover, we have also

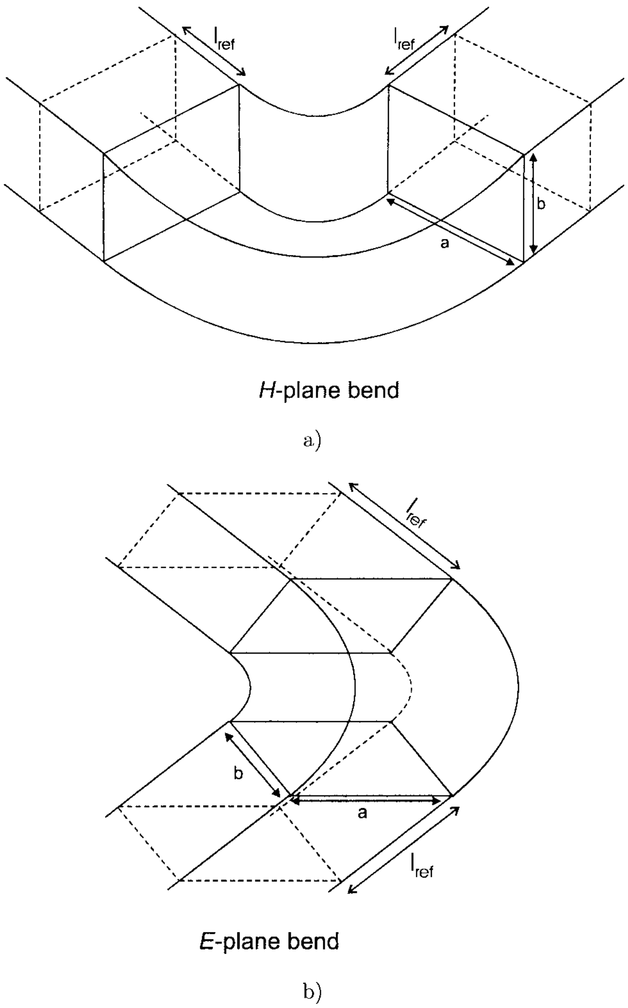


Fig. 1. (a) H-plane uniform bend. (b) E-plane uniform bend.

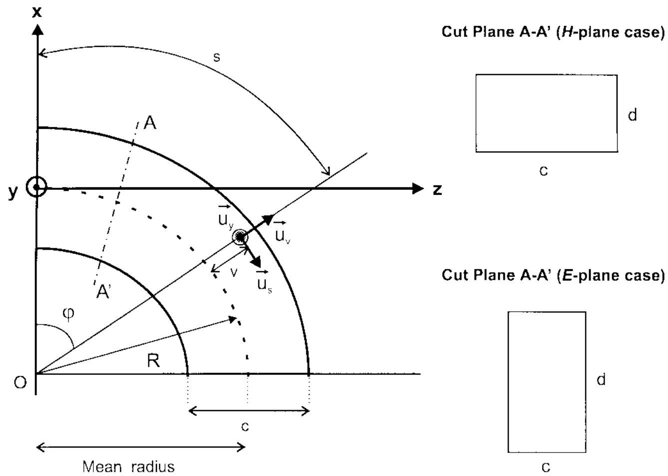


Fig. 2. Reference system used in the description of the curved waveguides.

investigated the case of a direct connection between an H- and an E-plane bend. Although this example is not very complex, we can consider it as representative of a great number of practical applications, whose accurate analysis is not possible considering only the excitation of the fundamental mode. Therefore, the main contributions of this paper are the calculation in a very efficient way of the full modal spectrum of curved regions, as well as the analysis of connections of bends taking under consideration any arbitrary incidence due to the excitation of the fundamental and higher order modes.

This paper is organized in two main sections. Section II is dedicated to describe the theoretical base of the problem under consideration. Thus, in Section II-A, a full-wave characterization of uniform circular bends is performed and the whole set of modes of the curved region is finally derived. Next, a rigorous analysis of junctions between straight and curved waveguides is presented in Section II-B. All mathematical details required to fully characterize the discontinuities considered can be found in Appendixes A and B. Finally, in Section III, the convergence of the proposed method is first discussed, and comparisons between our simulations and theoretical and experimental data are presented in order to fully validate the new analysis method proposed.

II. Theory

The first objective here is to obtain a full-wave modal solution of circular uniform bends. Next, making use of this multimodal characterization, junctions between straight and curved waveguide regions will be investigated in order to obtain the scattering parameters of the proposed structures.

A. Full-Wave Characterization of Uniform Bends in Rectangular Waveguide

Here, we will construct the whole set of modes of a continuously curved waveguide. Therefore, we must previously define the reference coordinate system that we are going to employ in order to describe the structures under investigation. After that, the Helmholtz equation for curved waveguides will be derived, and the TE^y_{mn} and TM^y_{mn} modes for the curved region will be constructed. The orthonormality condition that must satisfy these modes will then be analytically proven and, finally, the Helmholtz equation will be solved by means of the well-known method of moments.

1) Reference System Employed in the Description of Curved Waveguides: The reference system shown in Fig. 2 and defined by the set of coordinates (v, y, s), which was proposed by Lewin *et al.* [5], has been chosen to analyze the uniform curved waveguides. In this figure, c and d are the transversal dimensions of the curved waveguide, R denotes the mean radius of curvature of the bend, and O represents the center of curvature of the waveguide. It is important to point up that $s = R \cdot \varphi$ defines the direction of propagation, and the v coordinate establishes the displacement with regard to the mean radius R. With reference to Fig. 2, it must be emphasized that the c dimension determines the curvature plane of the bend and it must be suitably chosen when analyzing H- and E-plane bends. Concretely, for a given rectangular cross section of the bend of dimensions a and b ($a > b$), we must choose $c = a$ and $d = b$ for the analysis of H-plane bends, and for the analysis of E-plane bends, we must choose $c = b$ and $d = a$. Mathematical details of this new reference system can be found in Appendix A.

2) Helmholtz Equation for Uniform Bends in Rectangular Waveguide: Construction of the Modes in the Curved Region: We start with the following equations relating the electromagnetic fields $\vec{E}$ and $\vec{H}$ with the electric and magnetic vector potentials $\vec{F}$ and $\vec{A}$, respectively [16]:

$$\vec{E} = -\nabla \times \vec{F} + \frac{1}{j\omega\epsilon}\,\nabla \times \nabla \times \vec{A} \tag{1}$$

$$\vec{H} = \nabla \times \vec{A} + \frac{1}{j\omega\mu}\,\nabla \times \nabla \times \vec{F} \tag{2}$$

where a time–harmonic dependence ($e^{j\omega t}$) has been assumed, and $\vec{F}$ and $\vec{A}$ satisfy the well-known Helmholtz equation

$$(\nabla^2 + \omega^2\mu\epsilon)\cdot \begin{Bmatrix} \vec{A} \\ \vec{F} \end{Bmatrix} = 0. \tag{3}$$

Following [2], we are going to construct the TE^y and TM^y modes in the curved region. Equation (3) can be rewritten as follows:

$$\left(\nabla^2 + \omega^2\mu\epsilon\right)\cdot \psi(v,\, y,\, s) = 0 \tag{4}$$

where $\psi(v,\, y,\, s)$ denotes the component along the y-axis of the $\vec{A}$ or $\vec{F}$ potential functions. Now, by using (34) collected in Appendix A, we can rewrite (4) as

$$\left(\xi^2\,\frac{\partial^2}{\partial v^2} + \xi^2\,\frac{\partial^2}{\partial y^2} + \xi\,\frac{\partial}{\partial v} + R^2\,\frac{\partial^2}{\partial s^2} + \xi^2\omega^2\mu\epsilon\right)\psi(v,\, y,\, s) = 0 \tag{5}$$

where $\xi(v) \equiv R + v$. Applying separation of variables on $\psi(v,\, y,\, s)$, and assuming a propagation type $e^{-j\beta s}$ for the modes advancing in the $s > 0$ direction, we can express the scalar potential as follows:

$$\psi(v,\, y,\, s) = f(v)\cdot g(y)\cdot e^{-j\beta s} \tag{6}$$

where $f(v)$ is an unknown function and $g(y)$ can be written as

$$g(y) = \begin{cases} \sin\left(\dfrac{n\pi}{d}\left(y + \dfrac{d}{2}\right)\right), & n = 1,\, 2,\, 3,\, \ldots \\[2ex] \cos\left(\dfrac{n\pi}{d}\left(y + \dfrac{d}{2}\right)\right), & n = 0,\, 1,\, 2,\, \ldots \end{cases} \tag{7}$$

due to the uniformity of the bend along the y-axis direction. Finally, after using the two previous equations, (5) becomes the following linear eingenvalue problem:

$$\left[\xi^2\,\frac{d^2}{dv^2} + \xi\,\frac{d}{dv} + \xi^2\left(\omega^2\mu\epsilon - \left(\frac{n\pi}{d}\right)^2\right)\right]\cdot f(v) = (R^2\beta^2)\cdot f(v) \tag{8}$$

which is the Helmholtz equation particularized to uniform rectangular waveguide bends. It should be noted that this equation applies to both TE^y and TM^y modes. Details of the solution of this equation, which is easily solved by means of Galerkin procedure, can be found in [11].

For the TM^y modes, the vector potential functions $\vec{F}$ and $\vec{A}$ must be chosen as indicated in [16]

$$\vec{F} = 0 \quad \vec{A} = \psi^{\mathrm{TM}^y}(v,\, y,\, s)\vec{u}_y. \tag{9}$$

Since $\psi^{\mathrm{TM}^y}(v,\, y,\, s)$ must satisfy the Dirichlet boundary conditions on the cross section of the curved waveguide, we get from (6) and (7)

$$\psi^{\mathrm{TM}^y}(v,\, y,\, s) = f^{\mathrm{TM}^y}(v)\cdot\cos\left(\frac{n\pi}{d}\left(y + \frac{d}{2}\right)\right)\cdot e^{-j\beta s},\quad n = 0,\, 1,\, 2,\, \ldots \tag{10}$$

where $f^{\mathrm{TM}^y}(v)$ is an unknown function determined through the method of moments. By substituting (9) into (1) and (2), it is possible to obtain the expression of the transverse electric and magnetic fields related to the TM^y modes in terms of the scalar modal function $\psi^{\mathrm{TM}^y}(v,\, y,\, s)$. Once the transverse-field components have been derived, we can obtain the following expressions for the normalized TM^y vector-mode functions:

$$\vec{e}_p^{\,\mathrm{TM}^y} = \frac{\dfrac{n\pi}{d}}{\omega^2\mu\epsilon - \left(\dfrac{n\pi}{d}\right)^2}\cdot\mathcal{N}_p^{\mathrm{TM}^y}\cdot\sin\left(\frac{n\pi}{d}\left(y + \frac{d}{2}\right)\right)\cdot\frac{df_m^{\mathrm{TM}^y}}{dv}\,\vec{u}_v - \mathcal{N}_p^{\mathrm{TM}^y}\cdot\cos\left(\frac{n\pi}{d}\left(y + \frac{d}{2}\right)\right)\cdot f_m^{\mathrm{TM}^y}(v)\vec{u}_y \tag{11}$$

$$\vec{h}_p^{\,\mathrm{TM}^y} = \frac{1}{1 + \frac{v}{R}}\cdot\mathcal{N}_p^{\mathrm{TM}^y}\cdot f_m^{\mathrm{TM}^y}(v)\cdot\cos\left(\frac{n\pi}{d}\left(y + \frac{d}{2}\right)\right)\vec{u}_v \tag{12}$$

where p is an arbitrary TM^y mode, whose modal indexes are $(m,\, n)$, and $\mathcal{N}_p^{\mathrm{TM}^y}$ represents the normalization constant of each vector-mode function in the rectangular cross section of dimensions c and d, whose expression can be easily deduced from Appendix B as follows:

$$\mathcal{N}_p^{\mathrm{TM}^y} = \left[\frac{d}{\epsilon_n}\cdot\int_{-c/2}^{c/2}\frac{1}{1 + \frac{v}{R}}\cdot\left(f_m^{\mathrm{TM}^y}(v)\right)^2\,dv\right]^{-1/2}. \tag{13}$$

In order to construct the TE^y modes in the curved region, the same procedure employed before with the TM^y modes can be followed, thus finally leading to the following expressions for the normalized TE^y vector-mode functions:

$$\vec{e}_p^{\,\mathrm{TE}^y} = \frac{1}{1 + \dfrac{v}{R}}\cdot\mathcal{N}_p^{\mathrm{TE}^y}\cdot\sin\left(\frac{n\pi}{d}\left(y + \frac{d}{2}\right)\right)\cdot f_m^{\mathrm{TE}^y}(v)\vec{u}_v \tag{14}$$

$$\vec{h}_p^{\,\mathrm{TE}^y} = \frac{\left(\dfrac{n\pi}{d}\right)}{\omega^2\mu\epsilon - \left(\dfrac{n\pi}{d}\right)^2}\cdot\mathcal{N}_p^{\mathrm{TE}^y}\cdot\cos\left(\frac{n\pi}{d}\left(y + \frac{d}{2}\right)\right)\cdot\frac{df_m^{\mathrm{TE}^y}}{dv}\,\vec{u}_v + \mathcal{N}_p^{\mathrm{TE}^y}\cdot\sin\left(\frac{n\pi}{d}\left(y + \frac{d}{2}\right)\right)\cdot f_m^{\mathrm{TE}^y}(v)\vec{u}_y \tag{15}$$

where $f^{\mathrm{TE}^y}(v)$ is a new unknown function and where $\mathcal{N}_p^{\mathrm{TE}^y}$ denotes the normalization constant of TE^y modes in the rectangular cross section of the curved waveguide, which can be expressed as shown in Appendix B as follows:

$$\mathcal{N}_p^{\mathrm{TE}^y} = \left[\frac{d}{2}\cdot\int_{-c/2}^{c/2} \frac{1}{1+\dfrac{v}{R}} \cdot \left(f_m^{\mathrm{TE}^y}(v)\right)^2 \, dv\right]^{-1/2} \tag{16}$$

being p an arbitrary TE^y mode of modal indexes (m, n).

Next, following [10], we make use of the just deduced complete set of vector-mode functions in order to expand any arbitrary transverse electromagnetic field in the curved region

$$\vec{E}_t = \sum_i V_i \cdot \vec{e}_i \quad \vec{H}_t = \sum_i I_i \cdot \vec{h}_i \tag{17}$$

where $\vec{e}_i$ and $\vec{h}_i$ are the normalized vector-mode functions related to the ith mode in the curved region, and where V_i and I_i are the modal voltages and currents representing the amplitudes of the corresponding vector-mode functions. Moreover, this expansion allows us to define a modal admittance related to the ith TE^y or TM^y mode as

$$Y_i^{\mathrm{TE}^y} = \frac{\omega^2\mu\epsilon - \left(\dfrac{n\pi}{d}\right)^2}{\omega\beta\mu} \quad Y_i^{\mathrm{TM}^y} = \frac{\omega\epsilon\beta}{\omega^2\mu\epsilon - \left(\dfrac{n\pi}{d}\right)^2}. \tag{18}$$

Once all the expressions for the normalized vector-mode functions have been calculated, the orthogonality relationship satisfied by the TE^y and TM^y modes in the curved region is verified in Appendix B, where the following general orthonormalization condition for such modes is also imposed:

$$\int\int_S \left(\vec{e}_p \times \vec{h}_q^*\right) \cdot \vec{u}_s \, dS_s = \delta_{pq}. \tag{19}$$

In this expression, p and q represent both TE^y and TM^y modes of the curved region.

B. Analysis of Discontinuities Between Straight and Curved Waveguides of Rectangular Cross Section

The key point in the analysis of uniform bends in rectangular waveguide lies in the transition from the straight waveguide to the curved waveguide region. Therefore, here, a rigorous analysis of junctions between straight and curved waveguides of rectangular cross section including the full set of modes previously derived is presented. The analysis procedure, based on the general network theory proposed in [14], evaluates a GAM representation of each planar junction under consideration. The method efficiently describes the interactions between all the excited modes in both sides of the analyzed junction by means of simple expressions that facilitate the software implementation. The GAM formulation for junctions between straight and curved waveguides can be readily obtained by correctly adapting the method described in [14] to this particular case.

1) Analysis of Bends in Rectangular Waveguide: In this section, a full-wave characterization of single and cascaded uniform bends in rectangular waveguide is presented. The key feature of this procedure is that it starts from the GAM representation of each element of the structure once they have been previously computed. Therefore, an arbitrary device involving bends can be analyzed by cascading the corresponding coupling matrices, thus giving place to a global multimode representation.

a) H- and E-plane bends: In a general representation of an H- or an E-plane bend, we can observe three waveguides (the curved waveguide and the input and output straight waveguides) and two discontinuities between the curved segment and the input and output straight ports. Both curved and straight uniform waveguides also have their corresponding GAM representation. Consequently, by correctly cascading all the involved coupling matrices, we can get the global multimode representation of an H- or an E-plane bend, which can be efficiently solved as indicated in [15] to obtain the scattering parameters of the considered bend.

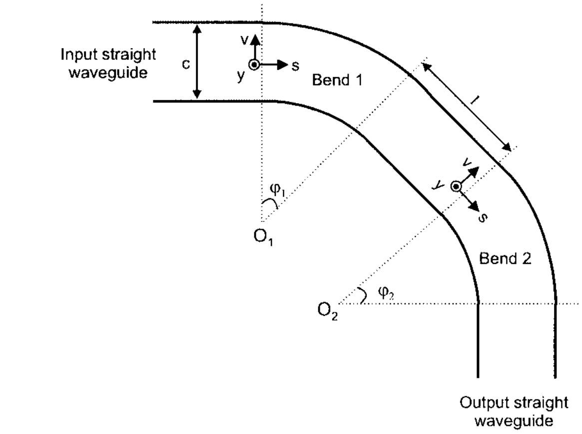
b) Connection of bends: Other structures involving several bends can be readily analyzed following the same procedure described above. The first structures considered are the so-called U and S configurations (see Fig. 3), both consisting essentially in connecting two E- or H-plane bends through a length l of straight waveguide. It is interesting to note that, in the S configuration, the involved bends have opposite curvature centers [see Fig. 3(b)]; therefore, each bend is described in a different reference system. Since the relationship between both reference systems can be easily set up as $v = -v'$, $y = -y'$, and $s = s'$, it is possible to obtain, after imposing the corresponding boundary conditions in the junction plane, the following relationship between the modal voltages and currents associated with the modes in both reference systems:

$$V_p = V_p'\left(\sin^2\left(\frac{m\pi}{2}\right) - \cos^2\left(\frac{m\pi}{2}\right)\right) \cdot \left(\sin^2\left(\frac{n\pi}{2}\right) - \cos^2\left(\frac{n\pi}{2}\right)\right) \tag{20}$$

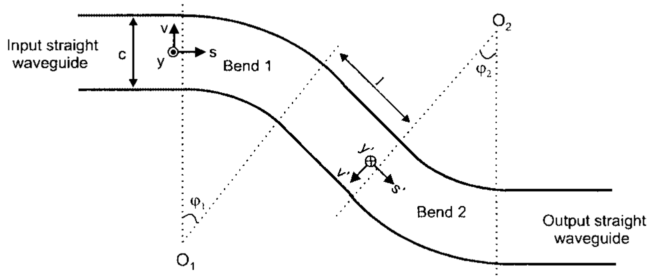
$$I_p = I_p'\left(\sin^2\left(\frac{m\pi}{2}\right) - \cos^2\left(\frac{m\pi}{2}\right)\right) \cdot \left(\sin^2\left(\frac{n\pi}{2}\right) - \cos^2\left(\frac{n\pi}{2}\right)\right) \tag{21}$$

where (m, n) are the modal indexes related to the pth mode of the straight waveguide between the two bends. Due to this analysis, we finally conclude that the classical GAM representation of the straight waveguide of length l between the two bends should be modified in order to account for the cited change of the reference systems. Thus, if $Y_{\mathrm{uw}}^{(\delta,\gamma)}$ $(\delta, \gamma = 1, 2)$ represents the well-known multimode admittance matrix representation of a length l of uniform waveguide [17], it must be rewritten now as follows:

$$\hat{Y}_{\mathrm{uw}}^{(\delta,\gamma)} = \begin{bmatrix} \hat{Y}_{\mathrm{uw}}^{(1,1)} & \hat{Y}_{\mathrm{uw}}^{(1,2)} \\ \hat{Y}_{\mathrm{uw}}^{(2,1)} & \hat{Y}_{\mathrm{uw}}^{(2,2)} \end{bmatrix} = \begin{bmatrix} Y_{\mathrm{uw}}^{(1,1)} & Y_{\mathrm{uw}}^{(1,2)} \cdot F \\ Y_{\mathrm{uw}}^{(2,1)} \cdot F & Y_{\mathrm{uw}}^{(2,2)} \end{bmatrix} \tag{22}$$



a)



b)

Fig. 3. Connection of bends. (a) U configuration. (b) S configuration (O_1 and O_2 are the centers of curvature of the involved bends).

where F is a diagonal matrix that relates both reference systems defined by the elements

$$F_{ij} = \left(\sin^2\left(\frac{m\pi}{2}\right) - \cos^2\left(\frac{m\pi}{2}\right) \right) \cdot \left(\sin^2\left(\frac{n\pi}{2}\right) - \cos^2\left(\frac{n\pi}{2}\right) \right) \cdot \delta_{ij} \quad (23)$$

where i and j point to the several modes considered in the GAM representation of the straight waveguide.

The last structure to be studied is a connection of an H-plane and an E-plane bend through a length l of a straight waveguide, as shown in Fig. 4. Following classical techniques [2]–[7], the analysis of this structure could be performed taking into account only the set of modes excited by each bend. For the H-plane bend case, and considering the reference system shown in Fig. 2, only the TM^y_{m0} modes should be considered at the reference planes, while the E-plane case should include the set of TE^y_{m1} modes. If the length l between the bends is large enough, the two bends can be connected, considering only the fundamental mode as an accessible mode, since higher order modes can be seen as localized modes (they attenuate very rapidly with the distance). On the contrary, if l is very small (or even zero), all the previous set of modes should be considered as excitation of both bends and, therefore, a complete family of modes must be taken into account in the solution of each bend. Following [2], we will consider in this case the TE^y_{mn} and TM^y_{mn} modes.

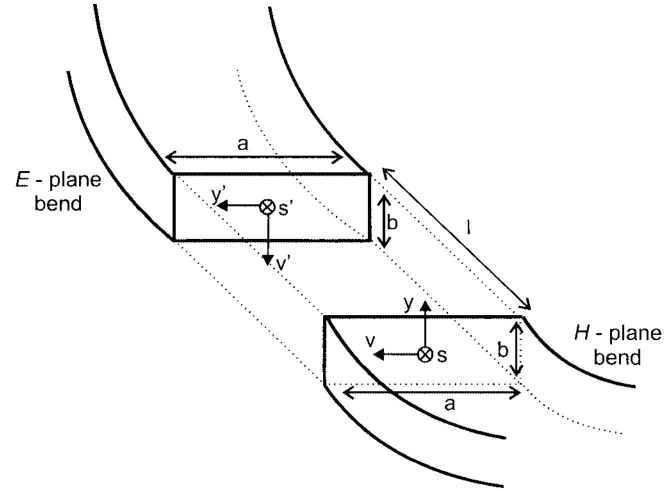


Fig. 4. Connection of an H- and E-plane bend.

As in the S configuration, a change of the reference systems that describe the two bends connected is also present in this case. As we can check from Fig. 4, the relationship between both reference systems is now $v = y'$, $y = -v'$, and $s = s'$. Following the same steps carried out in the discussion of the S configuration, it is then possible to conclude that this change of the reference system will modify the classical GAM $Y_{\mathrm{uw}}^{(\delta,\gamma)}$ ($\delta, \gamma = 1, 2$) of the straight waveguide placed between the bends so that its GAM will be now expressed as

$$\hat{Y}_{\mathrm{uw}}^{(\delta,\gamma)} = \begin{bmatrix} \hat{Y}_{\mathrm{uw}}^{(1,1)} & \hat{Y}_{\mathrm{uw}}^{(1,2)} \\ \hat{Y}_{\mathrm{uw}}^{(2,1)} & \hat{Y}_{\mathrm{uw}}^{(2,2)} \end{bmatrix} = \begin{bmatrix} Y_{\mathrm{uw}}^{(1,1)} & Y_{\mathrm{uw}}^{(1,2)} \cdot G \\ Y_{\mathrm{uw}}^{(2,1)} \cdot G & Y_{\mathrm{uw}}^{(2,2)} \end{bmatrix} \quad (24)$$

with G again being a diagonal matrix defined by the following elements:

$$G_{ij} = \Gamma_i \cdot \left(\cos^2\left(\frac{m\pi}{2}\right) - \sin^2\left(\frac{m\pi}{2}\right) \right) \cdot \delta_{ij} \quad (25)$$

where i and j denote the modes considered in the GAM formulation of the straight waveguide, m is the modal subindex related to the v' coordinate of the ith or jth mode, and where Γ_i is defined as follows:

$$\Gamma_i = \begin{cases} 1, & \text{if } i \text{ is a TE}^z \text{ mode of the rectangular waveguide} \\ -1, & \text{if } i \text{ is a TM}^z \text{ mode of the rectangular waveguide.} \end{cases} \quad (26)$$

III. Numerical and Experimental Results

Firstly, we focus our attention on the convergence of the propagation constants of the modes in the curved region to study the efficiency of the Galerkin procedure used in order to solve the Helmholtz equation obtained for curved waveguides [11]. In Fig. 5, we have represented the magnitude of the propagation constants related to the $\mathrm{TE}^y_{3,n}$ modes of a circular H-plane bend ($R = 15.24$ mm, $c = 22.90$ mm, $d = 10.20$ mm,

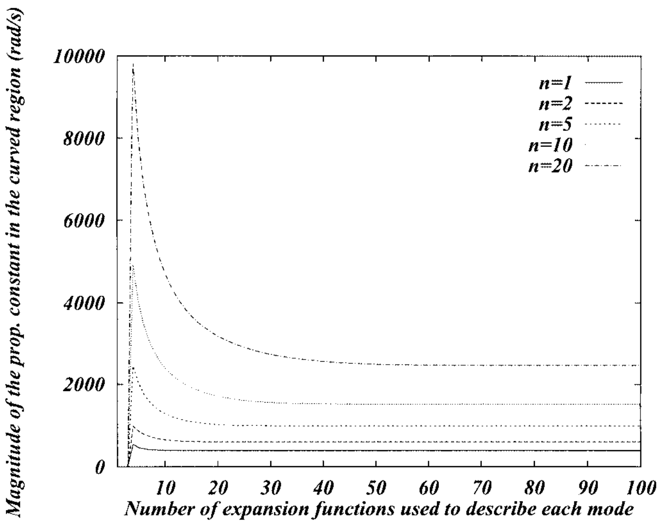


Fig. 5. Convergence of the propagation constant of the $\mathrm{TE}^{y}_{3,\,n}$ modes in the curved waveguide region ($n = 1,\ 2,\ 5,\ 10,\ 20$) as a function of the number of expansion functions used to describe each mode in the curved region (H-plane bend in WR-90 waveguide, $a = 22.90$ mm, $b = 10.20$ mm, $R = 15.24$ mm, frequency $= 10$ GHz).

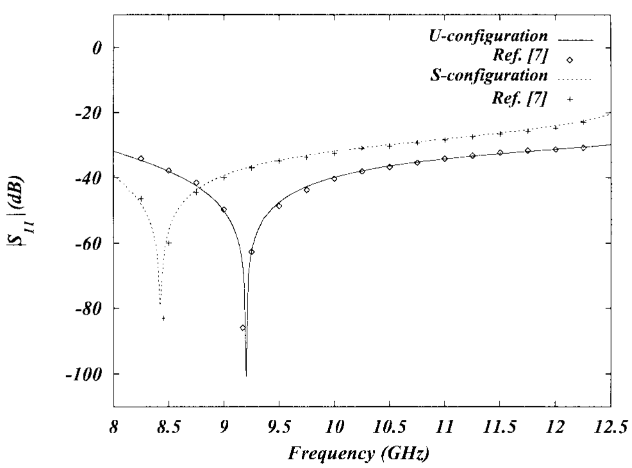


Fig. 6. Comparison between our results and numerical data from [7]. Both U and S configurations are analyzed. Cascaded 30° H-plane bends ($R = 15.24$ mm) through a straight transmission line of length $l = 5$ mm (WR-90 waveguide, $a = 22.90$ mm, $b = 10.20$ mm).

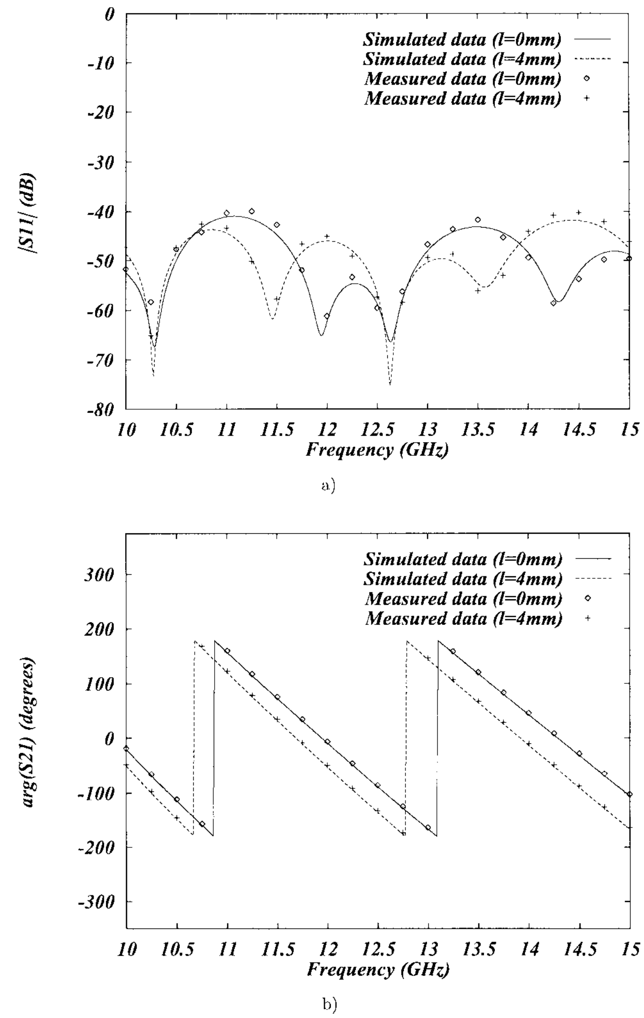


Fig. 7. Comparison between simulated and measured data. Connection of a 90° H-plane bend ($R = 31.25$ mm) and a 90° E-plane bend ($R = 31.25$ mm) in WR-75 waveguide. A straight transmission line of length l is included between the bends. (a) Magnitude of the reflection coefficient. (b) Phase of the transmission coefficient.

frequency$=$ 10 GHz) with $n = 1$, 2, 5, 10, 20 as a function of the number of expansion functions used in the description of $f^{\mathrm{TM}^y}(v)$ and $f^{\mathrm{TE}^y}(v)$ [see (11) and (14)]. By analyzing this figure, one observes that higher order modes need more expansion functions than lower ones in order to reach convergent results. However, a high order mode like the $\mathrm{TE}^{y}_{3,\,20}$ only needs 50 terms to achieve convergence. Consequently, we conclude that the new method we have proposed to evaluate the full spectrum of the modes in curved regions is very efficient from the computational point-of-view.

Next, in Fig. 6 our results for the U and S configurations are compared with the ones obtained in [7]. Once again, our simulated results are in excellent agreement with the numerical data available in the literature, thus validating our analysis method.

Junctions between H- and E-plane bends are then investigated with regard to authors' measurements. In Fig. 7, the magnitude of the reflection coefficient and the phase of the transmission coefficient for a junction between a 90° H-plane bend and a 90° E-plane bend in WR-75 waveguide are presented. It is important to point out again that this new structure could not be analyzed by means of classical methods [2]–[7] since it excites the complete set of TM^{y}_{mn} and TE^{y}_{mn} modes. In Fig. 7, two cases are analyzed: in the first one, we do not include a straight transmission line between both bends ($l = 0$ mm) and, in the second case, $l = 4$ mm. Our results are successfully validated by comparisons with the measured data. It should be emphasized that the computation time for a typical analysis (with 30 modes in the network and 20 basis functions in order to describe each mode in the curved region) is 0.228 s/frequency point on a Pentium III (933 MHz) personal computer. Therefore, the developed software becomes ideal to be employed into CAD software packages of complex waveguide systems involving uniform bends.

IV. Conclusion

A rigorous and computationally efficient method for analyzing uniform curved bends in rectangular waveguide has been proposed, considering the possible incidence of any arbitrary mode of the rectangular input port. The discontinuities between straight and curved waveguide regions have been analyzed by means of the GAM representation, thus providing a full-wave characterization of the investigated structures. The convergence of the presented method has been discussed and comparisons of our simulated results with numerical and experimental data have fully validated the new proposed theory. In conclusion, the analysis method described can be easily inserted into CAD tools of complex passive microwave devices for space and ground applications.

APPENDIX A
REFERENCE SYSTEM FOR DESCRIBING CURVED WAVEGUIDES

The relation between the coordinate system (v, y, s) and the associated Cartesian coordinate system (x, y, z) can be easily derived from Fig. 2 as follows:

$$x = R \cdot \left[-1 + \left(1 + \frac{v}{R}\right) \cdot \cos\left(\frac{s}{R}\right)\right] \tag{27}$$

$$y = y \tag{28}$$

$$z = R \cdot \left(1 + \frac{v}{R}\right) \cdot \sin\left(\frac{s}{R}\right). \tag{29}$$

On the other hand, the unit vectors can be also easily obtained as follows:

$$\begin{aligned} \vec{u}_v &= \cos\varphi \vec{u}_x + \sin\varphi \vec{u}_z \\ \vec{u}_y &= \vec{u}_y \\ \vec{u}_s &= -\sin\varphi \vec{u}_x + \cos\varphi \vec{u}_z. \end{aligned} \tag{30}$$

The unit vectors of the coordinate system have been also represented in Fig. 2. As we can verify from this figure, $\vec{u}_s$ defines the direction of propagation in the curved waveguide region and $\vec{u}_v$ moves away from the center of curvature. Finally, if f denotes an arbitrary scalar function and $\vec{A}$ denotes a vectorial function, the differential operators related to the reference system chosen can be easily calculated in the following way:

$$\nabla f(v, y, s) = \frac{\partial f}{\partial v}\vec{u}_v + \frac{\partial f}{\partial y}\vec{u}_y + \frac{1}{1+\frac{v}{R}}\frac{\partial f}{\partial s}\vec{u}_s \tag{31}$$

$$\nabla \cdot \vec{A}(v, y, s) = \frac{1}{1+\frac{v}{R}} \cdot \left[\frac{\partial}{\partial v}\left(A_v \cdot \left(1+\frac{v}{R}\right)\right) + \frac{\partial}{\partial y}\left(A_y \cdot \left(1+\frac{v}{R}\right)\right) + \frac{\partial A_s}{\partial s}\right] \tag{32}$$

$$\nabla \times \vec{A}(v, y, s) = \frac{1}{1+\frac{v}{R}} \cdot \begin{vmatrix} \vec{u}_v & \vec{u}_y & \left(1+\frac{v}{R}\right)\vec{u}_s \\ \frac{\partial}{\partial v} & \frac{\partial}{\partial y} & \frac{\partial}{\partial s} \\ A_v & A_y & \left(1+\frac{v}{R}\right) \cdot A_s \end{vmatrix} \tag{33}$$

$$\nabla^2 f = \frac{1}{R+v}\frac{\partial f}{\partial v} + \frac{\partial^2 f}{\partial v^2} + \frac{\partial^2 f}{\partial y^2} + \frac{1}{\left(1+\frac{v}{R}\right)^2}\frac{\partial^2 f}{\partial s^2}. \tag{34}$$

APPENDIX B
ORTHONORMALITY RELATIONSHIP BETWEEN THE MODES IN THE CURVED REGION

We begin defining an orthogonality condition between two arbitrary TM^y modes: the pth of subindexes (m_1, n_1) and the qth of subindexes (m_2, n_2) [the superscript TM^y is omitted in the functions $f(v)$ and $\psi(v, y)$] as follows:

$$\psi_p(v, y, s) = f_{m_1}(v) \cdot \cos\left(\frac{n_1\pi}{d}\left(y + \frac{d}{2}\right)\right) \cdot e^{-j\beta_p s} \tag{35}$$

$$\psi_q(v, y, s) = f_{m_2}(v) \cdot \cos\left(\frac{n_2\pi}{d}\left(y + \frac{d}{2}\right)\right) \cdot e^{-j\beta_q s}. \tag{36}$$

Equation (8) yields for each of the above modes

$$\xi^2 \frac{d^2 f_{m_1}}{dv^2} + \xi \frac{df_{m_1}}{dv} + \xi^2 \left(\omega^2\mu\epsilon - \left(\frac{n_1\pi}{d}\right)^2\right) \cdot f_{m_1}(v) - R^2\beta_p^2 \cdot f_{m_1}(v) = 0 \tag{37}$$

$$\xi^2 \frac{d^2 f_{m_2}}{dv^2} + \xi \frac{df_{m_2}}{dv} + \xi^2 \left(\omega^2\mu\epsilon - \left(\frac{n_2\pi}{d}\right)^2\right) \cdot f_{m_2}(v) - R^2\beta_q^2 \cdot f_{m_2}(v) = 0. \tag{38}$$

If we multiply (37) by $(f_{m_2}(v)/\xi)$ and (38) by $(f_{m_1}(v)/\xi)$, and then subtract the two resulting equations, we get the following expression:

$$\frac{d}{dv}\left[\xi\left(\frac{df_{m_1}}{dv}f_{m_2} - \frac{df_{m_2}}{dv}f_{m_1}\right)\right] + \xi\left(\left(\frac{n_2\pi}{d}\right)^2 - \left(\frac{n_1\pi}{d}\right)^2\right) \cdot f_{m_1} \cdot f_{m_2} = R^2\left(\beta_p^2 - \beta_q^2\right)\frac{1}{\xi} \cdot f_{m_1} \cdot f_{m_2}. \tag{39}$$

The discussion requires to distinguish between two cases. In the first one, we will assume that $n_1 = n_2$ and $m_1 \neq m_2$. Thus, if p and q represent two nondegenerate modes, and taking into account Dirichlet boundary conditions, one easily concludes from (39) that

$$\int_{-c/2}^{c/2} f_{m_1}(v) \cdot f_{m_2}(v) \cdot \frac{1}{1+\frac{v}{R}}\, dv = 0. \tag{40}$$

When $n_1 \neq n_2$ (independently of m_1 and m_2), we find that the cosine functions related to the scalar functions ψ_p and ψ_q are mutually orthogonal. Consequently, we finally deduce that the desired orthogonality condition between two arbitrary nondegenerate TM^y modes p and q should be mathematically expressed by means of the following integral in terms of the modal scalar function $\psi(v, y)$:

$$\int\int_S \psi_p(v,y)\cdot\psi_q(v,y)\cdot\frac{1}{1+\frac{v}{R}}\,dS_s = \frac{d}{\epsilon_{n_1}}\cdot\delta_{n_1,n_2}\cdot\int_{-c/2}^{c/2} f_{m_1}(v)\cdot f_{m_2}(v)\cdot\frac{1}{1+\frac{v}{R}}\,dv = \begin{cases} 0, & \text{if } n_1\neq n_2 \\ 0, & \text{if } n_1=n_2 \text{ and } m_1\neq m_2 \\ \neq 0, & \text{if } n_1=n_2 \text{ and } m_1=m_2 \end{cases} \tag{41}$$

where δ_{n_1,n_2} represents the Kronecker delta and ϵ_{n_1} denotes the well-known Neumann function [10]. With reference to condition (41), it should be noted that our interest, however, is to obtain an orthonormalization condition in terms of the vector-mode functions used in the expansion of an arbitrary TM^y electromagnetic field. Thus, a new orthonormalization condition can be readily obtained in terms of the normalized vector-mode functions calculated in Section II-A.2 as follows:

$$\int\int_S \left(\vec{e}_p^{\,\mathrm{TM}^y}\times\vec{h}_q^{\,\mathrm{TM}^y *}\right)\cdot\vec{u}_s\,dS_s = \delta_{pq} \tag{42}$$

which allows to determine (13) for the normalization constant $\mathcal{N}_p^{\mathrm{TM}^y}$ of these modes. For the TE^y modes, the same results obtained before can be easily derived following the same procedure. Therefore, we conclude that TE^y modes satisfy the same orthonormalization condition stated in (42) for TM^y modes.

Finally, the case in which p is a TE^y mode and q is a TM^y mode must be studied. Firstly, let $p(m_1, n_1)$ be an arbitrary TM^y mode and $q(m_2, n_2)$ be a TE^y one. Using (11) and (15), we can obtain

$$\int\int_S \left(\vec{e}_p^{\,\mathrm{TM}^y}\times\vec{h}_q^{\,\mathrm{TE}^y *}\right)\vec{u}_s\,dS_s = \mathcal{N}_p^{\mathrm{TM}^y}\mathcal{N}_q^{\mathrm{TE}^y}\cdot\left[\frac{\left(\frac{n_1\pi}{d}\right)}{\omega^2\mu\epsilon-\left(\frac{n_1\pi}{d}\right)^2}I_1I_2+\frac{\left(\frac{n_2\pi}{d}\right)}{\omega^2\mu\epsilon-\left(\frac{n_2\pi}{d}\right)^2}I_3I_4\right] \tag{43}$$

where the integrals I_1, I_2, I_3, and I_4 are defined as follows:

$$I_1\equiv\int_{-d/2}^{d/2}\sin\left(\frac{n_1\pi}{d}\left(y+\frac{d}{2}\right)\right)\cdot\sin\left(\frac{n_2\pi}{d}\left(y+\frac{d}{2}\right)\right)\,dy \tag{44}$$

$$I_2\equiv\int_{-c/2}^{c/2} f_{m_2}^{\mathrm{TE}^y}(v)\cdot\frac{df_{m_1}^{\mathrm{TM}^y}}{dv}\,dv \tag{45}$$

$$I_3\equiv\int_{-d/2}^{d/2}\cos\left(\frac{n_1\pi}{d}\left(y+\frac{d}{2}\right)\right)\cdot\cos\left(\frac{n_2\pi}{d}\left(y+\frac{d}{2}\right)\right)\,dy \tag{46}$$

$$I_4\equiv\int_{-c/2}^{c/2} f_{m_1}^{\mathrm{TM}^y}(v)\cdot\frac{df_{m_2}^{\mathrm{TE}^y}}{dv}\,dv. \tag{47}$$

The discussion of the above integrals requires to consider two cases. In the first one, we will consider that $n_1\neq n_2$. This assumption leads to $I_1=I_3=0$ and, consequently, integral (43) would be equal to zero. The next case is to assume $n_1=n_2$. Integral (43) now yields

$$\int\int_S \left(\vec{e}_p^{\,\mathrm{TM}^y}\times\vec{h}_q^{\,\mathrm{TE}^y *}\right)\cdot\vec{u}_s\,dS_s = \mathcal{N}_p^{\mathrm{TM}^y}\cdot\mathcal{N}_q^{\mathrm{TE}^y}\cdot\frac{\left(\frac{n_1\pi}{d}\right)}{\omega^2\mu\epsilon-\left(\frac{n_1\pi}{d}\right)^2}\cdot\frac{d}{2}\cdot\int_{-c/2}^{c/2}\frac{d\left(f_{m_1}^{\mathrm{TM}^y}f_{m_2}^{\mathrm{TE}^y}\right)}{dv}\,dv=0 \tag{48}$$

where we have taken into account the Dirichlet boundary condition.

On the other hand, let $p(m_1, n_1)$ be an arbitrary TE^y mode and $q(m_2, n_2)$ be an arbitrary TM^y mode. Considering now (12) and (14), we easily find that the two modes are mutually orthogonal for this trivial case.

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References

[1] S. O. Rice, "Reflections from circular bends in rectangular waveguides-matrix theory," *Bell Syst. Tech. J.*, vol. 27, no. 2, pp. 305–349, 1948.

[2] J. A. Cochran and R. G. Pecina, "Mode propagation in continuously curved waveguides," *Radio Sci.*, vol. 1 (new series), no. 6, pp. 679–696, June 1966.

[3] C. P. Bates, "Intermodal coupling at the junction between straight and curved waveguides," *Bell Syst. Tech. J.*, vol. 48, no. 7, pp. 2259–2280, Sept. 1969.

[4] R. Mittra *et al.*, *Computer Techniques for Electromagnetics*. London, U.K.: Hemisphere, 1987.

[5] L. Lewin, D. C. Chang, and E. F. Kuester, *Electromagnetic Waves and Curved Structures*. London, U.K.: Peregrinus, 1977.

[6] L. Accatino and G. Bertin, "Modal analysis of curved waveguides," in *20th Eur. Microwave Conf.*, Sept. 1990, pp. 1246–1250.

[7] A. Weisshaar, S. M. Goodnick, and V. K. Tripathi, "A rigorous and efficient method of moments solution for curved waveguide bends," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 2200–2206, Dec. 1992.

[8] M. Mongiardo, A. Morini, and T. Rozzi, "Analysis and design of full-band matched waveguide bends," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 2965–2971, Dec. 1995.

[9] P. L. Carle, "New accurate and simple equivalent circuit for circular E-plane bends in rectangular waveguide," *Electron. Lett.*, vol. 23, no. 10, pp. 531–532, May 1987.

[10] N. Marcuvitz, *Waveguide Handbook*, ser. IEE Electromagn. Waves 21. London, U.K.: Peregrinus, 1986.

[11] B. Gimeno and M. Guglielmi, "Multimode equivalent network representation for H- and E-plane bends in rectangular waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. 44, pp. 1679–1687, Oct. 1996.

[12] R. Pregla, "Concatenations of waveguide sections," *Proc. Inst. Elect. Eng.*, pt. H, vol. 144, no. 2, pp. 119–125, Apr. 1997.

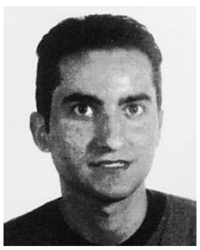
[13] P. Cornet, R. Dusséaux, and J. Chandezon, "Wave propagation in curved waveguides of rectangular cross section," *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp. 965–972, July 1999.

[14] A. Alvarez, G. Connor, and M. Guglielmi, "New simple procedure for the computation of the multimode admittance or impedance matrix of planar waveguide junctions," *IEEE Trans. Microwave Theory Tech.*, vol. 44, pp. 413–418, Mar. 1996.

[15] V. E. Boria, G. Gerini, and M. Guglielmi, "An efficient inversion technique for banded linear systems," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1997, pp. 1567–1570.

[16] R. Harrington, *Time–Harmonic Electromagnetic Fields*. New York: McGraw-Hill, 1961.

[17] R. E. Collin, *Field Theory of Guided Waves*, 2nd ed. New York: IEEE Press, 1991.



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